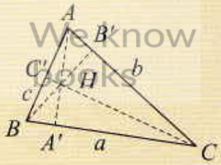
Arii și perimetre

• *Triunghiul.*



$A_{\Delta} = \dfrac{b \cdot h}{2}$; $P = a + b + c$;

$A_{\Delta} = \dfrac{BC \cdot AA'}{2} = \dfrac{AC \cdot BB'}{2} = \dfrac{AB \cdot CC'}{2}$;

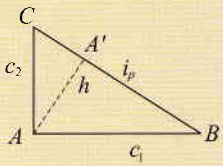
$A_{\Delta} = \dfrac{a \cdot b \cdot \sin\hat{C}}{2} = \dfrac{a \cdot c \cdot \sin\hat{B}}{2} = \dfrac{b \cdot c \cdot \sin\hat{A}}{2}$;

Formula lui Heron:

$A_{\Delta} = \sqrt{p(p-a)(p-b)(p-c)}$

cu $p = \dfrac{a+b+c}{2}$ (semiperimetrul)

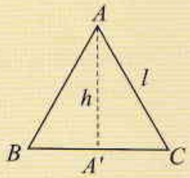
• *Triunghiul dreptunghic*



$A_{\Delta\ \text{dreptunghic}} = \dfrac{c_1 \cdot c_2}{2} = \dfrac{i_p \cdot h}{2}$;

$h_{ip} = \dfrac{c_1 \cdot c_2}{i_p}$

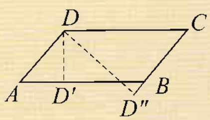
• *Triunghiul echilateral*



$A_{\Delta\ \text{echil}} = \dfrac{l^2\sqrt{3}}{4}$;

$h_{\Delta\ \text{echil}} = \dfrac{l\sqrt{3}}{2}$;

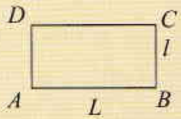
• *Paralelogramul*



$A_{\square} = b \cdot h = AB \cdot AD \cdot \sin\hat{A}$;

$P = 2AB + 2BC$;

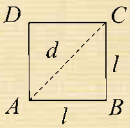
• *Dreptunghiul*



$A_{\square} = b \cdot h = L \cdot l$;

$P = 2L + 2l$;

• *Pătratul*



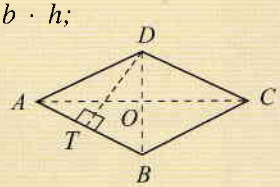
$A_{\square} = l^2$;

$d = l\sqrt{2}$ (diagonala pătratului);

$P = 4 \cdot l$;

• *Rombul*

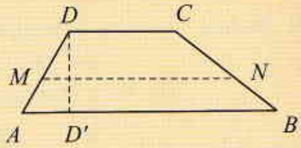
$A_{\diamond} = \dfrac{d_1 \cdot d_2}{2} = l^2\sin\hat{A} = b \cdot h$;



$P = 4 \cdot AD$;

$DT = h$

• *Trapezul*



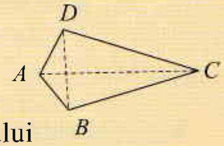
$A_{\text{trapez}} = \dfrac{(B+b) \cdot h}{2}$

MN linia mijlocie;

$A_{\text{trapez}} = MN \cdot h$

$P = AB + BC + CD + AD$;

• *Patrulater ortodiagonal* ($AC \perp BD$)

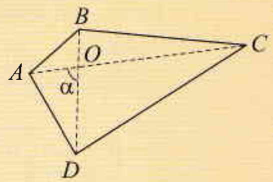


$A_{\text{patr. ortogonal}} = \dfrac{d_1 \cdot d_2}{2}$;

$P = AB + BC + CD + AD$;

d_1, d_2 diagonalele patrulaterului

• *Patrulater convex*

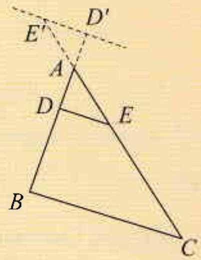


$A_{\text{patr. convex}} = \dfrac{d_1 \cdot d_2 \sin\alpha}{2}$;

$\alpha = m(\widehat{d_1, d_2})$;

Relații metrice în triunghi

* *Teorema lui Thales*



dacă $DE \parallel BC \Rightarrow \dfrac{DA}{DB} = \dfrac{EA}{EC}$

sau

dacă $D'E' \parallel BC \Rightarrow \dfrac{D'A}{D'B} = \dfrac{E'A}{E'C}$

* *Reciproca teoremei lui Thales*

dacă $\dfrac{DA}{DB} = \dfrac{EA}{EC}$ atunci $DE \parallel BC$

sau dacă $\dfrac{D'A}{D'B} = \dfrac{E'A}{E'C}$ atunci $D'E' \parallel BC$

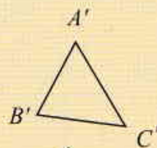
* *Teorema fundamentală a asemănării*

Dacă $DE \parallel BC$ atunci $\Delta ADE \sim \Delta ABC$

sau

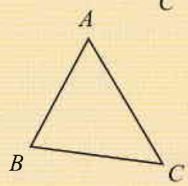
Dacă $D'E' \parallel BC$

atunci $\Delta AD'E' \sim \Delta ABC$



* *Cazuri de asemănare*

1) U.U



2) $\dfrac{AB}{A'B'} = \dfrac{AC}{A'C'}$ și $\hat{A} \equiv \hat{A}$

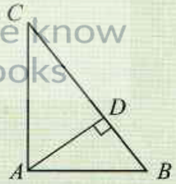
3) $\dfrac{AB}{A'B'} = \dfrac{AC}{A'C'} = \dfrac{BC}{B'C'}$

din 1) sau 2) sau 3) $\Rightarrow \Delta A'B'C' \sim \Delta ABC$

* *Teorema catetei*

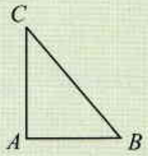
Dacă ΔABC

$$\left.\begin{array}{l} m(\hat{A}) = 90^\circ \\ Pr_{BC}AB = BD \\ Pr_{BC}AC = DC \end{array}\right\} \Rightarrow \begin{array}{l} AB^2 = BD \cdot BC \\ AC^2 = DC \cdot BC \end{array}$$



* *Teorema înălțimii*

$$\left.\begin{array}{l} \Delta ABC \\ m(\hat{A}) = 90^\circ \\ AD \perp BC \end{array}\right\} \Rightarrow AD^2 = BD \cdot DC$$



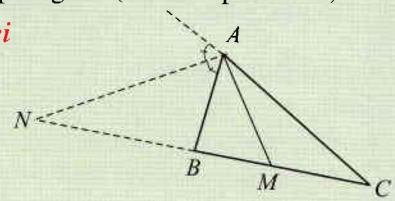
* *Teorema lui Pitagora*

$$\left.\begin{array}{l} \Delta ABC \\ m(\hat{A}) = 90^\circ \end{array}\right\} BC^2 = AB^2 + AC^2$$

* *Reciproca teoremei lui Pitagora*

Dacă în ΔABC avem $BC > AC > AB$ și $BC^2 = AC^2 + AB^2$ atunci ΔABC este dreptunghic (cu BC ipotenuza).

* *Teorema bisectoarei*



1) **interioare**

$$\left.\begin{array}{l} \Delta ABC \\ (AM \text{ bisectoarea internă}) \end{array}\right\} \Rightarrow \frac{BM}{MC} = \frac{AB}{AC}$$

2) **exterioare**

$$\left.\begin{array}{l} \Delta ABC \\ (AN \text{ bisectoarea exterioară}) \end{array}\right\} \Rightarrow \frac{NB}{NC} = \frac{AB}{AC}$$

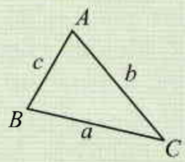
* *Într-un triunghi oarecare*

T_1 *Teorema cosinusului*

$a^2 = b^2 + c^2 - 2bc \cdot \cos\hat{A};$

$b^2 = a^2 + c^2 - 2ac \cdot \cos\hat{B};$

$c^2 = a^2 + b^2 - 2ab \cdot \cos\hat{C}.$

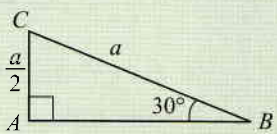


T_2 *Teorema sinusului*

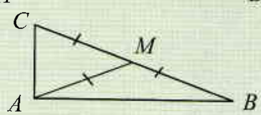
$$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}} = 2R.$$

R raza cercului circumscris triunghiului

* *Într-un triunghi dreptunghic*



T_3 Într-un triunghi dreptunghic cateta opusă unghiului de 30° este jumătate din ipotenuză.



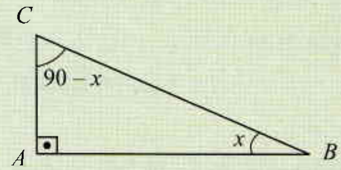
T_4 Într-un triunghi dreptunghic mediana dusă din vârful unghiului drept este jumătate din ipotenuză $AM = \dfrac{BC}{2}$.

• **Elemente de trigonometrie**

$$\sin x = \frac{\text{cateta opusă}}{\text{ipotenuză}} \qquad \cos x = \frac{\text{cateta alăturată}}{\text{ipotenuză}}$$

$$\text{ctg } x = \frac{\text{cateta alăturată}}{\text{cateta opusă}} \qquad \text{tg } x = \frac{\text{cateta opusă}}{\text{cateta alăturată}}$$

$\sin^2 x + \cos^2 x = 1$



$$\text{tg } x = \frac{\sin x}{\cos x};$$

$$\text{ctg } x = \frac{\cos x}{\sin x}; \ \text{tg } x = \frac{1}{\text{ctg } x} \qquad m(\hat{A}) = 90^\circ$$

$\sin x = \cos(90^\circ - x^\circ)$ $\qquad$ $\text{tg } x = \text{ctg}(90 - x^\circ)$

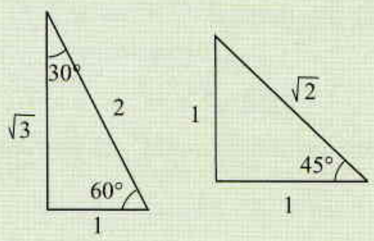
$\cos x = \sin(90^\circ - x^\circ)$ $\qquad$ $\text{ctg } x = \text{tg}(90 - x^\circ)$

$\sin(180^\circ - x^\circ) = \sin x$ $\qquad$ $\cos(180^\circ - x^\circ) = -\cos x^\circ$

	30°	45°	60°
sinus	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cosinus	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tangenta	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
cotangenta	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

Așa vei reține mai ușor

↓



$$\sin x = \frac{AC}{BC};\ \cos x = \frac{AB}{BC};\ \text{tg} x = \frac{AC}{AB};\ \text{ctg} x = \frac{AB}{AC}.$$